MA 591-003
Mathematical Foundations of Quantum Computation

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Course Description: Quantum computing and quantum information science are emerging disciplines in which the principles of quantum physics are employed to store and process information. Quantum technologies are pushing forward the frontiers of the future. At the time when quantum computation is fully developed, many salient applications will stand to benefit from this fast, concurrent, and secure information processing ability — we will be able to handle some of the most pressing problems the world faces as well as leap to discoveries not yet known. As such, students should be curious and find it beneficial and of vital importance that they are exposed to this subject as early as possible. A solid grasp and holistic treatment of this subject will require disciplines across multiple academic fields, which is hard to come by. This course intends to serve as the first step by introducing this subject from the mathematical perspective. Mathematical and computational foundations useful for deciphering the rich structure within a quantum system and the quantum computation will be discussed.

Prerequisites: A familiarity with linear algebra, e.g., inner product spaces, unitary transformations, is sufficient. Some familiarity with basic logic gates for Boolean functions, e.g., AND, XOR, and truth table, will be useful.

Course Structure:

1. The course will be delivered mainly via lectures.
   (a) The lecture begins with the seemingly easy but richly structured tensor product spaces over which some general concepts in quantum mechanics, e.g., pure state, density matrix, quantum entanglement, separability, and reversible operation, are described.
   (b) Then the notion of qubits, quantum gates, circuits, and the associated effect on computation will be explored in mathematical terms.
   (c) Some classical quantum algorithms will be re-examined, first mathematically, then in a quantum matter, to exemplify both the theory and the implementation.
2. Students will be asked to participate by completing homework assignments and possibly submitting projects on selected topics.

Learning Outcomes: This course aims at exposing students via the mathematical perspective how the quantum computation can be understood and formulated. Its goal is to build basic, and somewhat in-depth, mathematical knowledge needed for more advanced quantum computation.

Time and Location:

- TuTh 4:30PM - 5:45PM
- SAS 2225, North Campus

Course Materials:

1. (Textbook) Quantum Computing: From Linear Algebra to Physical Realization; Mikio Nakahara and Tetsuo Ohmi; CRC Press, 2008, ISBN 978-0-7503-0983-7.

2. (Reference book) Quantum Computer Science: An Introduction; N. David Mermin; Cambridge, 2007, ISBN 978-0-521-87658-2.

3. (Topical papers) A few selected but accessible articles in the literature.

4. Lecture notes will be made available online.

Topics to be covered:

- Tensor product space
  - Hilbert space over complex field
  - Representation/approximation via Schauder or Hamel bases
  - Tensor algebra and induced inner product
  - Linear transformations over tensor product space and their own tensor products
  - Schmidt decomposition vs. singular value decomposition
- Basic postulates in quantum mechanics
  - Pure vs. mixed states
  - Density matrix and its statistical meaning
  - Entanglement vs. separability
  - Trace preserving state transformation
  - Bipartite systems
- Basic quantum computing tools
  - Bits vs. qubits
  - Reversible operations of qubits, measurement and the Born rule
  - Basic logic gates vs. quantum gates
  - Circuit design in computation
  - Non-cloning theorem
- Computational examples:
  - Deutschs problem
  - Bernstein-Vazirani problem
  - Grover algorithm
  - Simons problem
  - Constructing the Toffoli gate
- Applications:
  - RSA encryption
  - Period finding and Shor factorization
  - Quantum Fourier transform.
  - Quantum algorithm for solving algebraic equations
  - Quantum algorithm for solving differential equations

Grading:

- Homework assignments (60%) and projects (40%).